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NUMERICAL MODELING OF A GAS-DISCHARGE CO_2 LASER WITH DIFFUSION COOLING

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A mathematical model of a CO_2 laser with diffusion cooling is proposed that is based on the simultaneous solution of glow-discharge equations in a quasilinear approximation with account for ambipolar diffusion, energy equations, and vibrational relaxation of the radiating gas. Consideration is given to a steady glow discharge in a cylindrical tube for the case of an electronegative gas. The model of a plane-parallel resonator is used on the assumption of constant radiation intensity inside the resonator. The developed model differs from the known ones by allowance for negative ions.

In [1], the existing gas-discharge CO_2 lasers with diffusion cooling were analyzed and experimental data were presented. There are different approaches to the problem of mathematical modeling of such lasers. The most simple of them is based on the assumption that the distribution of the specific energy contribution in the volume of a discharge-resonator chamber and the reduced strength of the electric field are known a priori. An obvious drawback of such a mathematical model is the impossibility of studying the interference of the energy characteristics of the laser and the parameters of the glow discharge. In [2], a method was proposed that relies on the simultaneous solution of glow-discharge equations and equations that describe a variation in the gasdynamic quantities. Within the framework of the proposed mathematical model, as in [2], the plasma is assumed to be quasineutral but negative ions are taken into account.

Basic Equations. The energy equation with account for the vibrational nonequilibrium for an axisymmetric case is of the following form [2]:

$$\frac{1}{r}\frac{d}{dr}\left(\frac{r\mu}{\mathrm{Pr}}\frac{dh}{dr}\right)+w-gI=0\,, \tag{1}$$

The gas enthalpy h is expressed by the relation

$$h=C_pT+e\,. \tag{2}$$

The Clapeyron equation of state holds for the gas. The pressure p is assumed to be constant over the cross section of the discharge tube. The Sutherland equation is used for the molecular viscosity. The value Pr = 0.7 is taken for the Prandtl number. A simplified single-mode model of the vibrational relaxation in a CO_2–N_2–He gas mixture [3] is used that was previously employed in [4]. On the assumption that $T_1 = T_2$ and $T_3 = T_4$, the kinetic model reduces to a single equation for ε_3:

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{\mu}{\mathrm{Sc}}\frac{d\varepsilon_3}{dr}\right)+\rho\omega+\frac{\delta w}{\theta_3R_g(\xi_1+\xi_2)}-\frac{gI}{\theta_3R_g(\xi_1+\xi_2)}=0\,,$$
$$\varepsilon_3=\left(\exp\frac{\theta_3}{T_3}-1\right)^{-1}, \tag{3}$$

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where T_j and θ_j are the vibrational and characteristic temperatures (j = 1, 2, 3, 4 corresponds to the symmetric, deformation, and asymmetric vibrational modes of CO_2 and to the vibrational mode of N_2). The internal energy of unit mass of the gas is specified by the equation

$$e = R_g (\xi_1 (\theta_1\varepsilon_1 + 2\theta_2\varepsilon_2 + \theta_3\varepsilon_3) + \xi_2\theta_4\varepsilon_4) , \quad \varepsilon_j = \left(\exp\frac{\theta_j}{T} - 1\right)^{-1} \quad (j \neq 3) .$$

The function ω is of the form (see, for example, [3])

$$\omega = \frac{\xi_1 K_{32}}{\xi_1 + \xi_2}\left[\varepsilon_2^3 (1 + \varepsilon_3) \exp\left(\frac{-500}{T}\right) - \varepsilon_3 (1 + \varepsilon_2)^3\right],$$

$$K_{32} = p\left(\frac{2.5\xi_1 + \xi_2}{44 - 865T^{-1/3} + 4500T^{-2/3}} + \xi_3 \exp (8.1 - 137T^{-1/3} + 426T^{-2/3})\right).$$

The value Sc = 0.7 is assumed for the Schmidt number. With the quasineutrality condition fulfilled, the spatial distribution of electrons and of positive and negative ions in a weakly ionized plasma is found from the balance equation on the assumption of ambipolar diffusion [5]:

$$\frac{d}{dr}\left(D_e^a \frac{dn_e}{dr}\right) + r (k_i N n_e - k_a N n_e - \beta_e n_e n_+ + k_d N n_-) = 0 , \tag{4}$$

$$\frac{d}{dr}\left(D_+^a \frac{dn_+}{dr}\right) + r (k_i N n_e - \beta_e n_e n_+ - \beta_i n_+ n_-) = 0 , \tag{5}$$

$$n_- = n_+ - n_e . \tag{6}$$

The electric-field strength E is determined from the condition that the total current J is constant along the discharge tube:

$$2\pi q \int_0^R n_e v_e r dr = J . \tag{7}$$

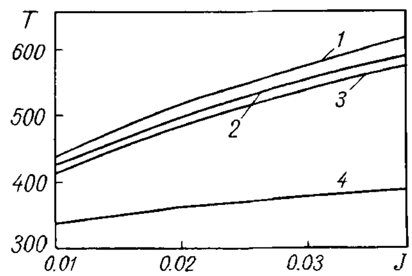
The specific volumetric energy contribution is calculated from the equation

$$w = E q v_e n_e . \tag{8}$$

The portion of energy that contributes to various degrees of freedom of molecules, the drift velocity of electrons, and the frequencies of ionization and attachment of electrons are determined with the aid of the energy distribution function of electrons that is obtained from solving the Boltzmann equation. The boundary conditions for unknown functions include the conditions on the tube axis r = 0 and at the tube wall r = R. On the tube axis, the conditions of symmetry are imposed:

$$\frac{dh}{dr} = 0 , \quad \frac{d\varepsilon_3}{dr} = 0 , \quad \frac{dn_e}{dr} = 0 , \quad \frac{dn_+}{dr} = 0 . \tag{9}$$

Two versions of the boundary condition for the wall temperature T_w are considered. In the first case it is assumed to be specified, and in the second case it is determined from the equation of heat transfer with the



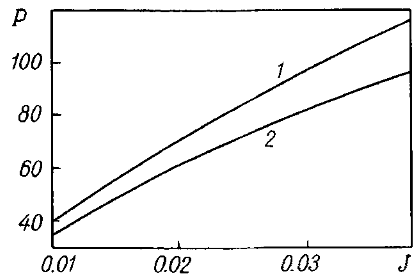


Fig. 1. Temperature T on the axis and at the wall of the discharge tube vs. discharge current J. T, K; J, A.

Fig. 2. Radiation power P vs. the discharge current J. P, W.

surrounding medium. When a horizontal tube is cooled, the gravitation-convection heat transfer is established, for which the coefficient of heat transfer with the air α can be expressed as [6]

$$\alpha = 3.4\left(\frac{T_0 - T_c}{R_1}\right)^{0.25}, \tag{10}$$

where T_0 and T_c are the external temperature of the wall and the temperature of the cooling medium, respectively, and R_1 is the outside radius of the tube (in m). The evaluations revealed that T_w and T_0 differ little; therefore T_w can be determined using the equation

$$T_w - T_c = \frac{Q}{2\pi\alpha R_1}. \tag{11}$$

The boundary conditions for n_e and n_+ at the wall $r = R$ are of the form

$$n_e = n_+ = 0. \tag{12}$$

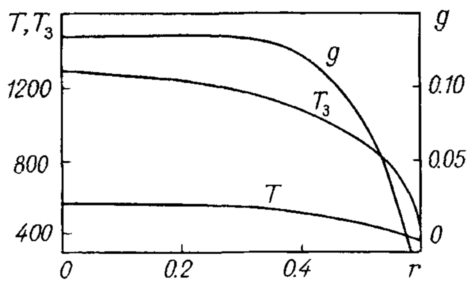
For T_3 at the wall, the condition of absolute catalyticity is used:

$$T_3 = T_w.$$

Moreover, the pressure p should be assigned (the pressure drop along the tube length is disregarded). The model of a plane-parallel resonator is used on the assumption of constant radiation intensity inside the resonator (see [3]).

Calculation Results. Differential equations (1) and (3)-(5) were solved numerically. Because the characteristics at the wall are strongly nonuniform, the transverse coordinate was transformed. Results of the numerical modeling by the proposed method were compared with data [7] for the laser with diffusion cooling, in which a $CO_2 : N_2 : He = 1 : 3 : 13$ mixture was used. For this laser, $L = L_0 = 140$ cm, the radius of the discharge tube was $R = 0.6$ cm, and the transmissivity of the output mirror was $\Theta = 0.2$. Figure 1 presents the temperatures T on the axis and at the wall of the discharge tube as functions of the discharge current. Curves 1 and 2 depict the temperature on the axis with and without account for radiation, respectively, for boundary condition (11), curve 3 is the temperature on the axis with account for radiation for the boundary condition $T_w = 300$ K, and curve 4 is T_w for boundary condition (11).

Figure 2 presents the radiation power P as a function of the discharge current for a specified wall temperature (curve 1) and boundary condition (11) (curve 2). Typical profiles of the gas temperature, vibrational temperature T_3, and amplification factors g are shown in Fig. 3. Figure 4 illustrates the charge distribution over the cross section of the discharge tube. The electron concentration is somewhat lower than the



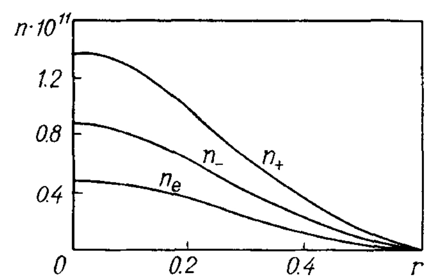


Fig. 3. Profiles of the gas temperature, vibrational temperature T_3, and amplification factor g. g, 1/m; r, cm.

Fig. 4. Charge distribution over the cross section of the discharge tube. n, 1/cm^3.

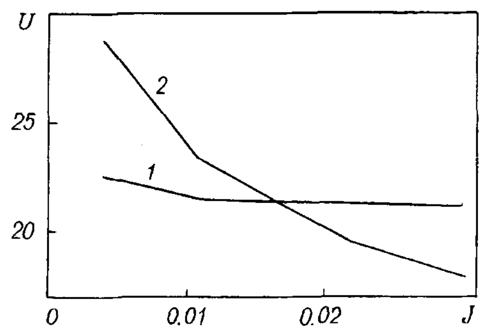


Fig. 5. Volt-ampere characteristics of the discharge: 1) calculation [2]; 2) present work. U, kV.

TABLE 1. Comparison of Experimental [7] and Calculation Values of the Generation Power P and Potential Difference U

p (torr)	J (A)	P_{exp} (W)	P_{calc} (W)	U_{exp} (V)	U_{calc} (V)
21	0.031	73	86.5	15.4	15.2
21	0.0345	72	93.3	14.8	14.8
20	0.036	75	93.2	15.0	14.2

TABLE 2. Comparison of Calculation Results with the Data of [2]

J (mA)	U (kV) [2]	U (kV) present work	n_e (cm^{-3}) [2]	n_e (cm^{-3}) present work	n_- (cm^{-3}) present work
4	22.5	28.8	$1\cdot10^{10}$	$1.1\cdot10^{10}$	4.6·1010
11	21.5	24.2	$3\cdot10^{10}$	3.1·1010	$7.2\cdot10^{10}$
22	21.3	20.3	$5.3\cdot10^{10}$	7·1010	$9\cdot10^{10}$

concentration of the negative ions. Table 1 presents a comparison of the calculation results with the experimental data of [7]. Note the good agreement of the results in the case where boundary condition (11) was used.

Also, a comparison with the calculation results of [2] was performed, in which a CO_2 : N_2 : He = 1 : 2 : 3 mixture was examined. In contrast to the mathematical model that is employed in the current work, the balance equation for negative ions was not considered in [2], and their role was taken into account indirectly, via introducing an effective recombination coefficient similarly to [5] by the equation

$$\beta_{ef} = \beta_e\left(1 + \frac{n_-}{n_e}\right).$$

Figure 5 presents volt-ampere characteristics (VAC) that were calculated using the given model and obtained in [2]. Attention is called to the fact that the VAC in [2] is sloping much less than the calculation results of the present work. To elucidate this fact, it is possible to refer to Table 2, which indicates that the ratio of the concentrations of the negative ions to the electron concentration on the axis of the discharge tube decreases with increase in the discharge current. The same character of the current dependence holds for β_{ef}. Therefore, a conclusion can be drawn that an adequate description of the discharge requires that the balance equation for negative ions be taken into account. In this connection, the VAC that was obtained in the present work seems to be more plausible than that calculated in [2].

NOTATION

C_p, specific heat at constant pressure; e, internal energy of unit mass of gas; R_g, gas constant; r, radial coordinate; ρ, density; p, pressure; T, temperature; h, gas enthalpy; Pr, Prandtl number; μ, gas viscosity; g, amplification factor; I, radiation intensity; Q, heat flux; R, tube radius; v_e, drift velocity of electrons; q, electron charge; n_e, n_+, and n_-, densities of electrons and of positive and negative ions; D_e^a, D_+^a, coefficients of ambipolar diffusion; V, volume density of gas molecules; k_i, k_a, and k_d, coefficients of impact ionization, attachment, and detachment; β_e, β_i, coefficients of electron-ion and ion-ion recombination; ξ_1, ξ_2, and ξ_3, molar concentrations of CO_2, N_2, and He; Sc, Schmidt number; δ, portion of discharge power that is admitted to the unified third-fourth vibrational mode.

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